Thick branes and 4D gravity

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Abstract: The induced 4d gravity on a brane world is analyzed. The case of a thick brane is considered and the unexpected appearance of a new threshold scale, much larger than the thickness scale is found. In cases of phenomenological interest, this new length scale turns out to be in the submillimeter range. The effect of R^2 corrections, both in the bulk and on the brane, is also studied. It is shown that they introduce new threshold scales and may induce drastic modifications to the leading behavior. A concrete string/D-brane realization of the induced gravity scenario is also presented.

Contents

1. Introduction

We consider a 3-brane in a higher-dimensional bulk as a model of our Universe. Gravity lives in the bulk, while the Standard Model particles are confined on the brane. Quantum loops of the latter will induce a four-dimensional Einstein term on the brane, which, as was pointed out in [1] for the case of a zero-thickness brane, can have surprising effects on the effective gravitational interaction on it. For a five-dimensional bulk, the relevant part of the action is parametrized as

$$S = M^3 \int d^5x \, \sqrt{-g}R + M^3 r_c \int d^4x \sqrt{-\hat{g}}\hat{R}, \tag{1.1}$$

where $\hat{g}_{\alpha\beta}$, with $\alpha, \beta = 0, 1, 2, 3$, is the induced metric on the 3-brane. When the fifth dimension is non-compact, the propagator of the graviton has the form $\sim M^{-3} \left(p + r_c p^2/2\right)^{-1}$, where p is the magnitude of its four-momentum along the brane. At length scales $l \sim p^{-1} \gg r_c$ gravity behaves as in five dimensions, while in the opposite limit $l \sim p^{-1} \ll r_c$ it is effectively four-dimensional.

It was further pointed out in [2] that, upon compactification of the fifth dimension on a circle of radius R with $R \ll r_c$, the gravitational interaction is four-dimensional at all scales[1]. This can be understood from the fact that gravity is four-dimensional at length scales smaller than r_c, while the compact circle makes spacetime effectively four-dimensional at length scales larger than R as well. There is an infinite discrete spectrum of graviton modes, with couplings to the brane fields that are suppressed compared to the usual compactification scenario. This modifies the appropriate experimental limits coming from solar system motions as well as energy loss in stars and supernovae, and allows values as exotic as $R \simeq 10^{-4} r_c \simeq 10^{16}$ m [2].

The purpose of this paper is to analyse the consistency of this scenario and its stability. There are several aspects that merit investigation:

a) The idealization of the brane as infinitely thin may not be a good approximation[2]. Any theory in which branes are intrinsic dynamical structures rather than external sources predicts a non-zero value for the brane thickness w. In such a theory there are (at least) two arbitrary parameters: M and R. The parameter r_c is determined by

[1] This may happen also without compactification, see [3].

[2] The effect of the brane thickness on gravity on the brane has also been studied in [5].

one-loop corrections to the Einstein term from 3-brane fields, while w is the thickness of the solitonic brane. We will evaluate r_c in the context of a string model later, and we expect w to be of order $1/M$[3]. However, for most of this work we will consider r_c and w as independent length scales that can be varied arbitrarily. Furthermore, we will assume that $w \ll R, r_c$. In the limit $w \to 0$ we should recover the results of [2]. Eventually, we would like to take w^{-1} to be the "near-new-physics" scale of ~ 10 TeV.

We are interested in physics at energies small compared to the thickness scale w^{-1}. At energies much larger than w^{-1}, the theory may be strongly coupled, or even not describable by field theory (as it may happen in string-theory examples). In section 2 we study the thick-brane situation with a non-compact fifth dimension. We find that the finite brane thickness produces an important modification to the $w = 0$ story: There is a new energy threshold appearing at $E_b = 1/\sqrt{wr_c}$. For energies $E_b \gg E \gg 1/r_c$ physics is four-dimensional. However, for $E \gg E_b$ the behavior depends strongly on the exact location on the brane along the fifth dimension. In particular, the gravitational potential between points at a distance $r \simeq 1/E$ along the brane becomes five-dimensional at the center of the brane.

At energies much smaller than the scale set by the brane thickness the detailed behavior across the brane cannot be resolved. For this reason, the physical observables involve averages along the fifth dimension. We study the behavior of such observables in general terms. We show that, at energies below the new threshold, their behavior deviates from four-dimensional. Moreover, the deviations are not universal for all sources localized on the brane. For an observer who views such sources as fundamental particles, this leads to violations of the equivalence principle (This was observed independently in [6].

In section 3 we study the case of a compact fifth dimension and analyse the graviton KK modes. Those with masses $m \ll E_b$ are almost constant across the brane and very similar to the ones studied in [2]. However, the modes with $m \gg E_b$ oscillate strongly between the two edges of the brane.

The compactified scenario is the only one consistent with the observed four-dimensional nature of gravity at distances larger than the solar system. For the allowed values of w, r_c and R, the new threshold appears at observable distances of the submillimeter range.

b) The emergence of a new energy scale that is not apparent in the action (1.1) raises the question of the importance of the terms omitted in it. The four-dimensional Einstein term is the first correction to the bulk action. The next corrections in a derivative expansion of the action involve the R^2 term in the bulk and the $\hat{R}^2$ term on the brane. In sections 4 and 5 we study the behavior of the propagator in their presence[4] terms was for one or more extra dimensions, and find that they also induce important modifications. The appearance of new threshold scales is generic in these

[3]This is not always necessary. We will see a different case in section 6.

[4]the graviton propagator in the presence of R^2 in the RS context was considered in [4].

enlarged theories.

We should mention at this point that, following [1], we assume the absence of cosmological constant terms both in the bulk and on the brane. This assumption represents two fine-tunings for which no justification is provided by the derivative expansion.

c) In order to have a theory with a controlled ultraviolet behavior where this scenario is realized, we consider in section 6 the low-energy limit of a string construction in which the evaluation of the R and R^2 terms is possible both on the brane and in the bulk. We demonstrate the appearance of a threshold scale at which the four-dimensional gravitational behavior breaks down. Moreover, we can predict in precise terms the new behavior.

Our conclusions are presented in the final section.

2. The non-compact case

We start with the 5d gravitational action

$$S_5 = M^3 \int d^5x \, \sqrt{-g} \; R. \tag{2.1}$$

In addition, we consider a brane, described by its collective coordinates $X^\mu(\xi)$ with $\mu = 0, 1, 2, 3, 4$, and carrying massless fluctuating fields. Their action is of the form

$$S_4 = T_4 \int d^4\xi \sqrt{-\hat{g}} L_{\rm matter}(\hat{g}, \phi_i), \tag{2.2}$$

where $\hat{g}_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$ is the induced metric, and ϕ_i denote collectively the other massless fields localized on the brane. We pick the static gauge $X^\alpha = \xi^\alpha$ and neglect the classical fluctuations of the brane ($X^4 \equiv z =$constant).

The action in (2.2) assumes an infinitely thin brane. When the brane has a non-zero thickness the brane fields have a finite extent in the fifth direction transverse to the brane. Let us denote the brane thickness by w. Apart from special situations, we expect that $w \sim 1/M$.

The quantum fluctuations of the brane fields renormalize the bulk action and in particular generate a correction to the five-dimensional Einstein term. Due to the unbroken four-dimensional Poincaré invariance, this is expected to be of the form

$$S_{\rm quantum} = M^3 r_c \int d^4x \, \sqrt{-\hat{g}} \; \hat{R}, \tag{2.3}$$

where the induced metric in the absence of fluctuations is $g_{\mu\nu}(x^\alpha, z = 0)$, and $\hat{R}$ is the four-dimensional curvature scalar constructed out of the induced metric.

In order to study the case of a brane of non-zero thickness we replace (2.3) by

$$S_{\rm quantum} = M^3 r_c \, \Delta(z) \int d^4x \, \sqrt{-\hat{g}} \; \hat{R}, \tag{2.4}$$

where $\Delta(z) = 1/w$ for $z \in [-w/2, w/2]$ and zero otherwise. The detailed structure of the brane and the resulting corrections to eq. (2.4) should be irrelevant, as long as we are interested in distances much larger than w. We will check to what extent this expectation is fulfilled by the solutions.

We compute the graviton propagator from the action $S_5 + S_{\rm quantum}$, as given by eqs. (2.2),(2.4). We ignore the tensor structure and solve instead for the scalar propagator $G(x^\alpha, z)$ (since this captures the essential behavior)

$$M^3 \left(\nabla_5^2 + r_c \Delta(z) \nabla_4^2\right) G(x^\alpha, z) = \delta^{(4)}(x^\alpha)\delta(z) \tag{2.5}$$

where ∇_n^2 is the flat Laplacian in n dimensions. Going to momentum space for the four-dimensional part we obtain the equivalent equation (we work in Euclidean space with $p^0 = -ip^5$)

$$M^3 \left(p^2 - \partial_z^2 + r_c \Delta(z) p^2\right) G(p^\alpha, z) = \delta(z), \tag{2.6}$$

where $p^2 = p_5^2 + p_1^2 + p_2^2 + p_3^2$. The solution is

$$G\left(p^\alpha, z\right) = A\ e^{-p|z|} \quad \text{for} \quad |z| > w/2 \tag{2.7}$$

and

$$G(p^\alpha, z) = B\ e^{\tilde{p}|z|} + C\ e^{-\tilde{p}|z|} \quad \text{for} \quad |z| < w/2, \tag{2.8}$$

with

$$\tilde{p} = \sqrt{1 + \frac{r_c}{w}}\, p \tag{2.9}$$

$$A = \frac{2\tilde{p}}{p + \tilde{p}} e^{\frac{1}{2}(p - \tilde{p})\, w}\ C, \qquad B = \frac{\tilde{p} - p}{\tilde{p} + p}\ e^{-\tilde{p}\, w}\ C \tag{2.10}$$

$$C = \frac{1}{2M^3}\ \frac{p + \tilde{p}}{\tilde{p}\left(p + \tilde{p} + (p - \tilde{p}) e^{-\tilde{p}\, w}\right)}. \tag{2.11}$$

In the limit $w \to 0$, $\Delta(z) \to \delta(z)$ we approach asymptotically the case of the infinitely thin brane analysed already in [2]. In this case $\tilde{p} \to \infty$ and $\tilde{p}\, w \to 0$, so that

$$A_0 = 2C_0 = 2B_0 = \frac{1}{M^3(2p + r_c p^2)}. \tag{2.12}$$

The propagator exhibits four-dimensional fall-off for $p \gg r_c^{-1}$ and five-dimensional fall-off for $p \ll r_c^{-1}$.

Going back to the non-zero thickness propagator we find that at the center of the brane ($z = 0$) it is given by

$$G(p^\alpha, 0) = B + C = \frac{1}{2M^3}\ \frac{p + \tilde{p}}{\tilde{p}\left(p + \tilde{p} + (p - \tilde{p}) e^{-\tilde{p}\, w}\right)} \left(\frac{\tilde{p} - p}{\tilde{p} + p}\ e^{-\tilde{p}\, w} + 1\right). \tag{2.13}$$

For $w \to 0$ this expression reduces to (2.12). In the region $\tilde{p}\, w \ll 1$ the propagator can be approximated very well by the $w = 0$ one. Thus, the previous analysis suffices. For $\tilde{p}\, w \gg 1$, which implies $p \gg (r_c w)^{-1/2} \equiv E_b$, the exponentials die-off and we obtain

$$G(p^\alpha, 0) \simeq \frac{1}{2M^3} \frac{1}{\tilde{p}} = \frac{1}{2M^3} \sqrt{\frac{w}{r_c}}\ \frac{1}{p} = \frac{1}{2M^3 E_b r_c} \frac{1}{p}. \tag{2.14}$$

The propagator again exhibits five-dimensional behavior with an effective Planck constant $M(r_c/w)^{1/6}$. Furthermore, a more thorough examination of the solution reveals that it has peculiar properties: Near the surface of the brane $G(p^\alpha, z)$ is exponentially suppressed by $\tilde{p}w/2$ when this is large. In fact, $G(p^\alpha, z)$ is significantly different from zero only within a region $|z| \lesssim \tilde{p}^{-1}$ around the center of the brane.

At energy scales much smaller than the scale set by the brane thickness the detailed z-dependence of the sources across the brane cannot be resolved. For this reason one must define effective correlations by averaging both the location of the source and the point of observation along the fifth dimension when computing the propagator. In order to achieve this we calculate the propagator with the δ-function located at some point in the interval $[-w/2, w/2]$. Then we average over the location of this point using a constant weight. Finally we average over the point of observation within the brane. A straightforward but lengthy calculation gives

$$\bar{G} = \frac{1}{M^3 r_c p^2}\left(1 - \frac{1}{\frac{p r_c}{2} + \frac{p}{E_b}\coth\left(\frac{p}{E_b}\right)}\right). \tag{2.15}$$

The averaged propagator displays behavior very similar to that of eq. (2.12) at all scales. For $p \ll E_b$ this is expected, as the solutions are almost constant across the brane. However, it is rather surprising for $p \gg E_b$. This result raises serious doubts on the physical relevance of the new scale E_b, as its observable effects seem to disappear.

In order to address this important question we consider the generalization of (2.6) for a source $J(z)$

$$M^3\left(p^2 - \partial_z^2 + r_c\Delta(z)p^2\right)G(p^\alpha, z) = J(z). \tag{2.16}$$

This is the equation of motion for the gravitational field, which we also denote by G. We assume that $J(z)$ is different from zero only within the brane boundaries and its integral in the region $[-w/2, w/2]$ is unity. By integrating both sides of this equation with respect to z in an interval $[-\delta/2, \delta/2]$, with δ a bit bigger than w, we obtain

$$\begin{aligned} M^3\left(\left.\frac{\partial G}{\partial z}\right|_{-\delta/2} - \left.\frac{\partial G}{\partial z}\right|_{\delta/2}\right) + wM^3p^2\int_{-\delta/2}^{\delta/2} G(p,z)\frac{dz}{w} \\ + r_c M^3 p^2 \int_{-w/2}^{w/2} G(p,z)\Delta(z)dz = 1. \end{aligned} \tag{2.17}$$

The first term in the above expression is negligible for $p \gg E_b$. For general smooth $\Delta(z)$ we expect a strong exponential fall-off of G with z, so that it becomes negligible outside the boundaries of the brane. The second term is always expected to be smaller than the third one for $w \ll r_c$ and smooth $\Delta(z)$. The conclusion is that

$$\int_{-w/2}^{w/2} G(p,z)\Delta(z)dz = \frac{1}{M^3 r_c p^2} \tag{2.18}$$

for $p \gg E_b$, to a very good approximation. Thus we confirm the physical picture implied by (2.15), and also generalize it to arbitrary smooth brane profiles $\Delta(z)$.

However, the physical quantity of interest is not the field averaged over the brane profile $\Delta(z)$, but over the profile $J'(z)$ of some probe. The distributions $\Delta(z)$ and $J(z)$ or $J'(z)$ are not identical, because $\Delta(z)$ results from loop corrections involving a multitude of intermediate states, each with a different profile. This fact has important phenomenological consequences: Only the propagator averaged over $\Delta(z)$ displays a pure four-dimensional behavior for $p \gg E_b$. Depending on the profile of the probe $J'(z)$ deviations from four-dimensional $\sim 1/r$ fall-off are expected. These can be substantial, as one may verify by evaluating the average of the solution (2.7)–(2.11) for various smooth functions $J'(z)$. Moreover, these deviations are not universal and, for an observer that considers the probes as fundamental particles, lead to violations of the equivalence principle. We conclude[5] that the new scale E_b has important phenomenological consequences. Similar arguments have been given in [6] for the question of charge universality when gauge fields are localized on the brane through a mechanism similar as well as for gravity.

The interpretation of these results must take into account the consistency of the whole picture. For $\tilde{p}\, w \ll 1$ the propagator is almost constant along the fifth dimension throughout the brane. Thus, we are within the region of validity of the low-energy description in terms of the action of eqs. (2.1), (2.4). For $\tilde{p}\, w \gg 1$ the main characteristic of the solution is the very strong variation of the propagator across the brane. Such behavior is associated with the high-energy regime, which we have assumed that has been integrated out in order to derive the low-energy action. This action seems to become insufficient for a consistent description at a scale $\sqrt{r_c w} \gg w$, contrary to our initial expectation. Thus the theory displays peculiar non-decoupling behavior.

In the following section we will see another manifestation of this behavior in the case of a compact fifth dimension. There, averaging the propagator corresponds to averaging strongly oscillatory KK modes on the brane. Therefore, it must be performed in a systematic fashion (through the use of the renormalization group for example) in order to have physical significance. Consequently the proper averaging depends on the (unknown) details of the fundamental theory.

The moral of this section is that theories with induced gravity on the brane exhibit a breakdown of predictability in the gravitational sector at an energy scale hierarchically lower than the fundamental one. In section 6 we will describe an example where the fundamental theory is controllable, and where we will be able to discern the physics beyond the new threshold scale.

[5]These conclusions do not depend on the shape of the thick-brane profile, as an analysis of other (smooth) profiles indicates.

3. Compactification

We compactify the fifth dimension on a circle of radius $R \gg w$. The compactification does not change the short-distance structure of the theory, while it turns the five-dimensional long-distance behavior into a four-dimensional one. In [2] R was taken a few orders of magnitude smaller than r_c, so that gravity is four-dimensional to a good approximation at all astrophysical scales.

The calculation of the propagator is completely analogous to the one in the previous section. The only difference is the use of periodic boundary conditions. We give the result for the interior of the brane, where

$$G(p^\alpha, z) = B\ e^{\tilde{p}|z|} + C\ e^{-\tilde{p}|z|}, \tag{3.1}$$

with

$$C = \frac{1}{2M^3\tilde{p}}\left[1 + \frac{p - \tilde{p}\coth\left(p\frac{R-w}{2}\right)}{p + \tilde{p}\coth\left(p\frac{R-w}{2}\right)} e^{-\tilde{p}w}\right]^{-1}, \tag{3.2}$$

$$B = C - \frac{1}{2M^3\tilde{p}}. \tag{3.3}$$

We are also interested in the spectrum of the KK modes resulting from this large-scale compactification. As before we ignore the tensorial structure and consider the case of a scalar field $\Phi(x^\alpha, z)$. The equation of motion for the KK modes is (we revert to Minkowski space)

$$\left(\partial_z^2 + m_n^2 + r_c\Delta(z)m_n^2\right)\phi_n(z) = 0, \tag{3.4}$$

where $\Phi(x^\alpha, z) = \sum_n \phi_n(z)\sigma_n(x^\alpha)$ and the $\sigma_n(x^\alpha)$ satisfy the four-dimensional Klein-Gordon equation $(\partial^\alpha\partial_\alpha + m_n^2)\sigma_n = 0$.

The solutions have the form (for simplicity we omit the index n from m_n, ϕ_n)

$$\phi(z) = A\ e^{i\tilde{m}z} + B\ e^{-i\tilde{m}z} \quad \text{for} \quad |z| < w/2, \tag{3.5}$$

$$\phi(z) = C\ e^{imz} + D\ e^{-imz} \quad \text{for} \quad w/2 < z < R - w/2, \tag{3.6}$$

with

$$\tilde{m} = \sqrt{1 + \frac{r_c}{w}}\, m. \tag{3.7}$$

The periodicity condition permits solutions only with discrete values of m. There are two classes of solutions, depending on the reflection properties of (3.6) around the center of the brane (at $z = 0$). The symmetric class has $A = B$, while the antisymmetric $A = -B$. The quantization conditions for the masses m are

$$\tan\left(m\frac{R-w}{2}\right) = -\left(\frac{\tilde{m}}{m}\right)^{\pm 1}\tan\left(\frac{\tilde{m}w}{2}\right), \tag{3.8}$$

with the $+(-)$ sign corresponding to the symmetric (antisymmetric) class. An overall amplitude of the solutions is determined through the normalization condition for the field.

For gravitational sources located at the center of the brane, such as the ones we are considering here, the antisymmetric solutions do not contribute to the propagator, which is a symmetric function of z. Thus, we concentrate on the symmetric sector. By switching to Minkowski space it can be checked that the propagator of eq. (3.1) has poles at the values given by eq. (3.8). In the limit $w \to 0$, the quantization condition gives

$$\tan\left(\frac{mR}{2}\right) = -\frac{mr_c}{2}, \tag{3.9}$$

in agreement with [2].

The behavior we observed in the previous section persists even after the compactification. For $p \ll E_b$ the propagator is almost constant across the brane and the results reproduce those of the scenario with an infinitely thin brane. For $p \gg E_b$ (or $\tilde{p}w \gg 1$) the propagator is exponentially suppressed, apart from a region $|z| \lesssim \tilde{p}^{-1}$ near the center of the brane. The strong variation of the propagator indicates that we are outside the region of validity of the low-energy description. Corrections originating in the specific structure of the brane become relevant not at a length scale $\sim w$ as expected, but at the much larger length scale $E_b^{-1} = \sqrt{wr_c}$.

The form of the KK modes confirms these conclusions: For $m \ll E_b$ the KK solutions are almost constant across the brane, while for $m \gg E_b$ they become strongly oscillatory. Averaging the propagator across the brane corresponds to averaging the strong fluctuations of the KK modes in the interior of the brane. The correct form of this procedure will be dictated by the fundamental theory.

As a final point we examine the suppression of the high KK modes relative to the zero mode on the brane. This is quantified by the ratio

$$\frac{|\phi_0(0)|^2}{|\phi_n(0)|^2} = \cos^2\frac{\tilde{m}w}{2}\left[\frac{1}{2}\left(1-\frac{w}{R}\right)\left(1+\frac{\tilde{m}^2}{m^2}\tan^2\frac{\tilde{m}w}{2}\right) - \frac{1}{mR}\frac{\tilde{m}}{m}\tan\frac{\tilde{m}w}{2}\right]$$

$$+\frac{w}{2R} + \frac{1}{2\tilde{m}R}\sin\tilde{m}w, \tag{3.10}$$

with $n > 0$. For $m \ll E_b$ one recovers the suppression factor $\sim m^2$ of [2], with the masses m being approximately $(2n+1)\pi/R$ and n a large integer. For $m \gg E_b$ the solution of eqs. (3.8),(3.10) indicates that there are unsuppressed modes with masses that are approximately $(2n+1)\pi E_b$. However, averaging these strongly oscillatory modes across the brane generates a suppression factor $\sim m$ for $\bar{\phi}$. As a result the averaged KK modes are still suppressed by $\sim m^2$ relative to the zero mode.

The compactified scenario is the only one that is phenomenologically viable [2]. In particular, R must be chosen much smaller than r_c, so that the gravitational interaction has four-dimensional behavior to a good approximation even at distances larger than the solar system. Reproducing correctly Newton's constant requires $M^3 r_c = M_{Pl}^2$. Assuming that the brane thickness is of order M (the natural choice) implies that the new threshold scale is

$$\frac{E_b^{-1}}{\text{mm}} \sim 2.4 \left(\frac{M}{\text{TeV}}\right)^{-2} = 5.6 \times 10^4 \left(\frac{r_c}{10^{20}\text{m}}\right)^{2/3}. \tag{3.11}$$

The parameter ranges allowed by observational constraints on planetary motion are: a) $R \gtrsim 10^{16}$m with $R/r_c = 10^{-4}$, and b) R smaller than the Earth radius and M in the TeV range. For the first range, the new threshold appears at a distance of several meters, a result that is in conflict with experiment. For the second range, if $M \sim 10$ TeV, then $E_b^{-1} \sim 20\mu$m.

4. The R^2 terms

In this section we consider the higher-order corrections in a derivative expansion of the action (1.1). Among these, the terms relevant for the graviton propagator are the R^2 terms in the bulk and on the brane. It is known that out of the three independent tensor structures, a linear combination (the Gauss-Bonnet term) does not contribute to the propagator. Our discussion thus, applies to the opposite case. As in the previous sections, we ignore the tensor structure and solve for the scalar propagator in the presence of four-derivative terms in the action. Moreover, we ignore related factors of order one that could appear in the equations of motion when they are derived through variation of the action. We assume the bulk to be five-dimensional.

First we consider the corrections localized on the 3-brane, which can be studied in the thin-brane limit. Let us parametrize them as

$$L_4 = \sqrt{-\hat{g}} \left(M^3 r_c \, \hat{R} + \lambda_2 \, \hat{R}^2\right). \tag{4.1}$$

The effective coefficients r_c, λ_2 include quantum corrections from the brane fields.

The propagator has the general form

$$G \sim \frac{1}{2p + r_c p^2 + \lambda_2 \, p^4/M^3}. \tag{4.2}$$

We are considering four-momenta $p \ll M$. In the regime $pr_c \ll 1$, we obtain a five-dimensional behavior provided that

$$\lambda_2 \ll (Mr_c)^3. \tag{4.3}$$

On the other hand, for $pr_c \gg 1$ we obtain a four-dimensional behavior provided that $Mr_c \geq 1$ (in particular, we are interested in $Mr_c \gg 1$) and

$$\frac{\lambda_2}{Mr_c} \left(\frac{p}{M}\right)^2 \ll 1. \tag{4.4}$$

If

$$\lambda_2 < Mr_c \tag{4.5}$$

the four-dimensional behavior extends up to the fundamental scale M. However, if $\lambda_2 \sim (Mr_c)^a$ with $1 \leq a \leq 3$, the four-dimensional behavior persists up to a scale $E'_b \sim M\ (Mr_c)^{(1-a)/2}$. This can be well below M (since by assumption $Mr_c \gg 1$). In particular, λ_2 can behave either as Mr_c or $(Mr_c)^2$. In the first case the four-dimensional behavior extends up to the scale M, while in the second there is a transition at $E'_b \sim \sqrt{M/r_c}$.

The R^2 corrections in the bulk are more delicate. A thin-brane approach, treating the bulk R^2 terms perturbatively, runs into problems with divergences, because squares of the δ-function appear. Therefore, it is obvious that a short-distance regularization is needed. The simplest way to implement one is to consider a thick brane, as in the previous sections.

Thus, we need to solve the equation

$$\left[p^2 - \partial_z^2 + L^2(p^2 - \partial_z^2)^2 + \Delta(z) r_c p^2\right] G = \delta(z), \tag{4.6}$$

where the length scale L controls the strength of the R^2 term and $\Delta(z)$ is defined in section 2. For $|z| > w/2$ the solution remains as before. For $|z| < w/2$ it has exactly the same form as before, but $\tilde{p}$ is now given by

$$\tilde{p} = \sqrt{p^2 + \frac{1 \pm \sqrt{1 - 4\frac{r_c}{w} L^2 p^2}}{2L^2}}. \tag{4.7}$$

We are interested in solutions that are finite in the limit $L \to 0$. This rules out the solution with the positive sign, which scales $\sim 1/L$ in this limit. The presence of a new scale $\sim \sqrt{w/r_c|L^2|}$ is apparent in this expression (L^2 can be positive or negative). For $p^2|L^2|r_c/w \ll 1$ we obtain

$$\tilde{p} = p\sqrt{1 + \frac{r_c}{w} + p^2 L^2 \frac{r_c^2}{w^2} + \ldots} \tag{4.8}$$

and for the propagator

$$G \sim \frac{1}{2p + r_c p^2(1 + \frac{r_c}{w} p^2 L^2 + \ldots)}. \tag{4.9}$$

In the (natural) case $|L^2| \sim w^2$, the new scale is the one found in the previous sections $\sim (wr_c)^{-1/2}$. For $|L^2|$ smaller than w^2, it shifts to larger energies. We see again that the detailed structure of the theory affects the low-energy behavior drastically. A simple power-counting argument would indicate that the R^2 correction is expected to be negligible relative to the leading R term at energies below $1/L$. However, for $w \ll r_c$ the new energy scale appears in this regime, and its exact value depends crucially on the coefficient of the R^2 term. This implies that the effect of higher terms in the derivative expansion is difficult to estimate in theories of induced gravity on a brane living in a higher-dimensional bulk. We will see similar examples in the following section.

5. More transverse dimensions

For future applications we would like to discuss the case with more than one large transverse dimensions [7]. We work in the thin-brane limit. Starting from

$$S = M^{n+2} \int d^{n+4}x\sqrt{-g}\; R_{n+4} + M^{n+2}r_c^n \int d^4x\sqrt{-\hat{g}}\; \hat{R}_4 \tag{5.1}$$

we obtain the propagator on the brane

$$G_n(p,0) \sim \frac{D_n(p,0)}{1+r_c^n p^2 D_n(p,0)} \;\;,\;\; D_n(p,0) = \int \frac{d^n q}{p^2+q^2}. \tag{5.2}$$

$D_n(p,0)$ is UV divergent. In order to define it we introduce a momentum cutoff Λ, which can be thought of as an alternative to the finite-thickness regularization of the previous section. Then

$$D_2(p,0) \sim \log[(\Lambda^2+p^2)/p^2] \;\;,\;\; D_4(p,0) \sim \Lambda^2 - p^2\log[(\Lambda^2+p^2)/p^2]. \tag{5.3}$$

For n=4 the propagator for momenta well below the cutoff is

$$G_4(p,0) \sim \frac{1}{p^2+\frac{1}{r_c^4\Lambda^2}}. \tag{5.4}$$

Thus, if $r_c\Lambda \gg 1$, for $\Lambda \gg p \gg (\Lambda r_c^2)^{-1}$ we have four-dimensional gravity, while for $p \ll (\Lambda r_c^2)^{-1}$ the gravitational force is screened.

Next we analyse the effect of bulk R^2 corrections in the higher-dimensional case. The perturbation in the action is parametrized by $M^{n+2}L^2\; R^2_{n+4}$. The solution for the four-dimensional propagator is given by (5.2), where now

$$D_n(p,0) = \int d^n k \frac{1}{(k^2+p^2+L^2(k^2+p^2)^2)}. \tag{5.5}$$

The appearance of the R^2 contribution in the propagator regulates the UV divergence for $n<4$.

For $n=2$ we obtain[6]

$$D_2(p,0) \sim -\log\left[1+\frac{1}{p^2L^2}\right]. \tag{5.6}$$

For $pL \ll 1$ and $pr_c \ll 1$ we obtain a propagator $\sim \log(p^2L^2)$. For $pL \ll 1$ but $pr_c \gg 1$ a four-dimensional behavior is obtained with $G_N \sim (M^4 r_c^2)^{-1}$. Finally, for $pL \gg 1$ we still have a four-dimensional behavior with a different Newton's constant, $G_N \sim (M^4(r_c^2-L^2))^{-1}$.

[6]In the following we omit factors of order one in the evaluation of $D_n(p,0)$. For this reason the estimates for the various threshold scales are only approximate.

For $n = 4$ D_4 is UV divergent. We introduce a cutoff and obtain

$$D_4(p,0) \sim p^2 \log\left[\frac{(p^2+\Lambda^2)L^2+1}{p^2L^2+1}\frac{p^2}{p^2+\Lambda^2}\right] + \frac{1}{L^2}\log\left[\frac{(p^2+\Lambda^2)L^2+1}{p^2L^2+1}\right]. \qquad (5.7)$$

As before, taking $\Lambda = \infty$ while keeping L fixed we obtain a uniform four-dimensional behavior. Another natural choice is that the cutoff scale determines the R^2 term: $\Lambda L \sim 1$. In this case, we find the same behavior as in the $L = 0$ case: a massive graviton with mass $\sim 1/\Lambda r_c^2$.

Clearly, there are more possibilities. We will always assume that $L\Lambda \gg 1$ and $r_c\Lambda \gg 1$. The approximate forms of $D_4(p,0)$ are

$$pL \ll 1, \qquad D_4 \sim \frac{2}{L^2}\log(L\Lambda) \qquad (5.8)$$

$$pL \gg 1 \text{ and } p \ll \Lambda, \qquad D_4 \sim \frac{1}{L^2}\log\frac{\Lambda^2}{p^2} \qquad (5.9)$$

$$pL \gg 1 \text{ and } p \gg \Lambda, \qquad D_4 \sim \frac{\Lambda^4}{2L^2p^4} \qquad (5.10)$$

There are three possible orderings of the various scales:
(i) $L/r_c^2 \ll 1/r_c \ll 1/L \ll \Lambda$: When $pL \ll 1$

$$G_4 \sim \frac{\log(L\Lambda)}{L^2 + 2p^2r_c^4\log(L\Lambda)}. \qquad (5.11)$$

When $p \ll L/r_c^2$ there is screening, while when $p \gg L/r_c^2$ we obtain $G_4 \sim p^{-2}$. For $1/L \ll p \ll \Lambda$ we still have $G_4 \sim p^{-2}$, while for $p \gg \Lambda$

$$G_4 \sim \frac{\Lambda^4}{(2L^2p^2 + r_c^4\Lambda^4)p^2}. \qquad (5.12)$$

For $\Lambda \ll p \ll r_c^2\Lambda^2/L$ we still have four-dimensional behavior, while for $p \gg r_c^2\Lambda^2/L$ we obtain $G_4 \sim \Lambda^4/(p^2L)^2$. To summarize, there is screening for $p \ll L/r_c^2$, $1/p^2$ behavior for $L/r_c^2 \ll p \ll r_c^2\Lambda^2/L$ and $1/p^4$ behavior above.
(ii) $1/L \ll 1/r_c \ll L/r_c^2 \ll \Lambda$: For $p \ll 1/L$ there is screening. For $L/r_c^2 \gg p \gg 1/L$ we find $G_4 \sim \log[p^2/\Lambda^2]/L^2$. When $\Lambda \gg p \gg L/r_c^2$, $G_4 \sim 1/p^2$. For $\Lambda^2r_c^2/L \gg p \gg \Lambda$ we still have $1/p^2$ behavior which turns to $1/p^4$ for $p \gg \Lambda^2r_c^2/L$.
(iii) $1/L \ll 1/r_c \ll \Lambda \ll L/r_c^2$: The situation for $p \ll \Lambda$ is as in case (ii). For $p \gg \Lambda$ we obtain $1/p^4$ behavior.

We conclude that on many occasions the R^2 terms can modify gravity substantially in various regimes introducing new threshold scales. The important observation is that these modifications may occur in energy regions where the higher-derivative terms such as R^2 are expected to be negligible. For example, in the case (i) for $n = 4$ discussed above the lower energy scale L/r_c^2 is much smaller than $1/L$. At such low energies the R^2 term should be completely negligible relative to the R term. However, the exact value of the new scale depends on the coefficient $1/L$ of the R^2 term.

There are also effects that have to do with the tensorial structure of gravity and the fact that R^2 terms affect this structure. These additional subtler effects eventually need to be taken into account as well, even though we will not do this here.

6. D-brane realization

String theory provides so far the unique example of a theory with a controllable quantum gravity (below the Planck scale) as well as ultraviolet-finite higher-dimensional theories (see [8] for introductory reviews). Moreover, it has provided us with calculable examples of brane solitons, namely D-branes, which are the concrete realizations of the brane-world idea.

We will consider here type-I vacua with N=2 supersymmetry. Realizations with less supersymmetry are possible but we will try to keep the structure as simple as possible. The general structure of such vacua is given by a collection of D9 and D5 branes embedded in a ten-dimensional bulk [9, 10]. We will consider two of the dimensions parallel to the branes to be compactified on a two-torus. The four extra dimensions are those of a non-compact R^4/Z_2 orbifold with the D5 branes located at the fixed point at the origin. Such vacua are decompactifications of four-dimensional N=2 type-I vacua with D5 branes when the K3 volume becomes infinite.

In such vacua there are eight infinite dimensions felt by the closed string sector (gravity) and D9 branes. The D5 branes (wrapped on T^2) have four infinite dimensions. Let T, U be the standard Kähler and complex structure moduli of T^2. For two orthogonal circles of radii $R_{1,2}$ they are given by $T = iR_1R_2$, $U = iR_1/R_2$. The following T-dualities are relevant:

$$R_2 \to 1/R_2 \Rightarrow T_2 \leftrightarrow U_2$$
$$R_1 \leftrightarrow R_2 \Rightarrow T_2 \to T_2 \quad , \quad U_2 \to 1/U_2 \tag{6.1}$$
$$R_1 \to 1/R_1 \Rightarrow T_2 \leftrightarrow 1/U_2$$

The relevant gravitational terms in the low energy effective action are

$$S = M^6 \int d^8x\ R_8 + M^6\ L^2 \int d^8x\ \ R_8^2 + ... + M^6 r_c^4 \int d^4x\ R_4 + \lambda_2 \int d^4x\ R_4^2 + ... \tag{6.2}$$

The Einstein term in eight dimensions comes from the sphere

$$M^6 = \frac{T_2}{g_s^2} M_s^6, \tag{6.3}$$

where M_s is the string scale, g_s is the (ten-dimensional) string coupling, T_2 the volume of the two torus, and we ignore factors of order one.

We now focus on the R_8^2 term. In the case of the type-I string in ten dimensions (N=4 supersymmetry) there is an R^2 contribution coming from the disk. Moreover it is known [11] that there are no further perturbative corrections to its effective coupling. There are however non-perturbative corrections to it coming from D5-brane instantons [12], that translate via heterotic/type-I duality to NS5-instantons on the heterotic side and via heterotic/type-II duality to fundamental string instantons on the type-II side [13, 14]. However, such corrections vanish in eight non-compact dimensions (they also vanish in the case where five dimensions are non-compact, that we will discuss below). The only other corrections to the R_8^2 terms may come from the projected D9 spectrum. These contributions can be obtained from heterotic duals [15] and can be shown to vanish in the decompactification limit. Thus,

$$M^6\ L^2 \sim \frac{T_2}{g_s} M_s^4 \Rightarrow L^2 \sim g_s\ M_s^{-2}. \tag{6.4}$$

The coefficient of the four-dimensional Einstein term can be inferred from the calculations in [16]. There, it was pointed out that, in type-I vacua of the type discussed here, there is a one-loop correction to the Einstein term. This matches via heterotic/type-I duality to the universal contribution to gauge couplings [17] as well as to the one-loop Kähler potential [18]. The calculations were carried out in four dimensions, but upon decompactication the contributions of the 9-5 and 5-5 sectors remain and give a purely four-dimensional contribution (originally six dimensional, it becomes four-dimensional due to the torus compactification). We obtain

$$M^6 r_c^4 \sim \frac{M_s^2}{T_2} E_2(U,\bar{U}) = \frac{M_s^2}{T_2} \sum_{(m,n)\in(Z,Z)-(0,0)} \frac{U_2^2}{|m+nU|^4}. \tag{6.5}$$

E_2 is modular invariant and has the following asymptotic expansion

$$E_2(U,\bar{U}) = 2\zeta(4)U_2^2 + \frac{\pi\zeta(3)}{U_2} + \mathcal{O}(e^{-\pi U_2}). \tag{6.6}$$

Finally, the R_4^2 term comes from the one-loop corrections of the 5-9 and the 5-5 states. The corrections are logarithmic and, extending the results of [16], we obtain

$$\lambda_2 \sim \log(T_2 U_2 |\eta(U)|^4). \tag{6.7}$$

We will further compactify three out of the four dimensions of the orbifold with radii of the order of the string scale. This may bring several D5 branes at finite distance of the order of the string scale, as there will be one at each fixed point of the orbifold. Other configurations are also possible with D5 branes being located at a single fixed point. In this case we do not have local tadpole cancellation, and non-trivial one-loop corrections to the tension can appear.

The effective action now becomes

$$S = M^3 \int d^5x\ R_5 + M^3\ L^2 \int d^5x\ \ R_5^2 + ... + M^3 r_c \int d^4x\ R_4 + \lambda_2 \int d^4x\ R_4^2 + ... \tag{6.8}$$

with

$$M^3 = \frac{T_2}{g_s^2} M_s^3 \ \ , \ \ M^3 \ L^2 \sim \frac{T_2}{g_s} M_s \tag{6.9}$$

$$M^3 r_c \sim \frac{M_s^2}{T_2} E_2(U, \bar{U}) \tag{6.10}$$

and λ_2 given in (6.7).

Thus we find $M r_c \sim g_s^{4/3} T_2^{-5/3} E_2(U, \bar{U})$, and $M_s r_c \sim E_2 g_s^2 / T_2^2$. Since we must be working in a weekly coupled description, which means that the volume should always be greater or equal to the string length we must consider the following inequivalent cases:

(1) $R_{1,2} \gg 1$ in string units, which implies that $T_2 >> 1$ and $E_2 \sim \mathcal{O}(1)$. In this case $M^3 r_c \sim 1/T_2$ and $\lambda_2 \sim \log(T_2)$.

(2) $R_1 \gg 1$, $R_2 \sim \mathcal{O}(1)$. In this case $T_2 \sim U_2 \gg 1$, $M^3 r_c \sim E_2(T_2)/T_2 \sim T_2$ and $\lambda_2 \sim T_2$.

(3) $R_1 \gg 1$ and $R_2 \ll 1$. In this case to go to a weakly coupled description we must T-dualize once, and we will be describing D4 branes with one transverse and one longitudinal direction large. In this case $T_2 \sim \mathcal{O}(1)$ and $U_2 \gg 1$. Upon T-duality $T_2 \leftrightarrow U_2$ with $T_2 \gg 1$ and $U_2 \sim \mathcal{O}(1)$. Then $M^3 r_c \sim E_2(T_2)/U_2 \sim T_2^2$ and $\lambda_2 \sim T_2$.

(4) $R_1 \sim R_2 \sim \mathcal{O}(1)$. Then $M^3 r_c \sim \mathcal{O}(1)$ and $\lambda_2 \sim \mathcal{O}(1)$.

(5) $R_{1,2} \ll 1$, so that $U_2 \sim \mathcal{O}(1)$, $T_2 \ll 1$. In this case we will T-dualize twice and obtain D3 branes with two large transverse dimensions. In the T-dual version $M^3 r_c \sim T_2 \gg 1$ and $\lambda_2 \sim \log(T_2)$.

From the various cases above only (3) can provide that $1/r_c$ is much smaller both from the string scale and the 5-dimensional Planck scale. In particular $M_s r_c \sim g_s^2 T_2 \gg 1$ and $M r_c \sim (g_s T_2)^{4/3} \gg 1$. Other relevant scales in this case are the compactification scale $M_c \sim M_s/\sqrt{T_2}$ which determines the decompactification of the two compact dimensions, the 5-d Planck scale $M_{(5)} \sim M_s g_s^{-2/3} T_2^{1/3}$, the eight dimensional Planck scale $M_{(8)} \sim M_s g_s^{-1/3} T_2^{1/6}$ as well as the ten-dimensional Planck scale $M_{(10)} \sim M_s g_s^{-1/4}$.

We are in the perturbative regime with $g_s < 1$ and $g_s T_2 \gg 1$. The ordering of the relevant scales is

$$1/r_c \ll M_c \ll M_s \ll M_{(10)} \ll M_{(8)} \ll M_{(5)} \tag{6.11}$$

At scales $E << 1/r_c$ gravity on the D4-brane is 5-dimensional. For $1/r_c \ll E \ll M_c$ gravity on D4 is four-dimensional. In the regime $M_c \ll E \ll M_s$ the two dimensions decompactify and physics (and gravity) in the bulk and on the D4 brane is seven-dimensional. For $M_s \ll E \ll M_{(10)}$ the other three compact directions decompactify. Moreover stringy effects become important. Thus, the bulk and brane physics is that of ten-dimensional non-compact string theory. Finally, for $E \gg M_{(10)}$ gravitational effects become strong and it is believed that the physics is dominated by ten-dimensional black-hole formation. Note that the $M_{(5)}$ and $M_{(*)}$ Planck scales are "mirage" scales since they do not correspond to physical thresholds of the theory.

Let us now analyse the R^2 corrections on the brane for case (3). In [19] it was shown that R^2 corrections on D-branes, unlike those in the bulk, do not come in the Gauss-Bonnet combination and are thus contributing to the graviton propagator. They become important for $E \sim M_s$ when also stringy effects become important. Note that this is much earlier than the five dimensional Planck scale. Moreover the bulk R^2 term according to our analysis in section 5, becomes important at the compactification scale M_c (this is compatible with an effective brane thickness $w \sim M_s^{-1}$ at energies below the winding threshold). However, the bulk R^2 terms in string theory are not expected to affect the graviton propagator since they come in the Gauss-Bonnet combination.

In the case where all orbifold directions are non-compact, our analysis in section 5 applies. Case (3) is the only one where the interesting threshold can be smaller than the string scale. Putting in numbers we obtain $L/r_c^2 \sim (g_s T_2)^{-1/2} M_s$, $1/r_c \sim (g_s^2 T_2)^{-1/4} M_s$ and $1/L \sim g_s^{-1/2} M_s$. We are clearly in case (i) of section 5. However all these scales are larger than the compactification scale $M_c \sim M_s/\sqrt{T_2}$ so the four-transverse dimensional discussion does not apply beyond M_c. We learn though that gravity on the D4-brane is screened up to the compactification scale, or for lengths larger than $\sqrt{T_2}$. Beyond M_c we have effectively a D4 brane in 10 non-compact dimensions. Thus, beyond M_c gravity is five-dimensional on the D4-brane.

7. Conclusions

The purpose of this paper was to provide a detailed analysis of the behavior of theories with gravity induced on a brane through quantum corrections by brane matter fields. We have relaxed various approximations made in previous studies [1, 2].

The first such approximation was the zero thickness of the brane. We considered thick four-dimensional branes of thickness w in a bulk. The natural choice is $w \sim 1/M$, with M the fundamental scale of the theory. We found that a new characteristic scale emerges in this case: $E_b^{-1} = \sqrt{w r_c}$, where $r_c \gg 1/M$ controls the strength of the induced Einstein term on the brane. At distances larger than the new scale the behavior in unaffected. However, at shorter distances it deviates strongly from the expected four-dimensional behavior. The solutions for the propagator and the KK modes vary strongly within the brane, and we calculated their averages between the edges. We found that the averaged gravitational potential displays deviations from the four-dimensional form $\sim 1/r$ which depend on the profile of the probe sources. This leads to violations of the equivalence principle.

The main conclusion to be drawn from the above is that the description in terms of the effective action loses its predictability at energies much smaller than the fundamental scale M. Only the knowledge of the structure of the fundamental theory can predict correctly the behavior at energies larger than the new threshold scale E_b.

For the phenomenologically viable scenario in the case of a compact fifth dimension the threshold is at experimentally accessible distances of the submillimeter range.

We also considered the next corrections in the derivative expansion of the effective action: the R^2 terms in the bulk and on the brane. The analysis for bulks with one or more extra dimensions strengthens the conclusion reached above: The emergence of new threshold scales is generic. Several examples were discussed in sections 5 and 6. Moreover, the new scales may appear in energy regimes where the effect of the higher-derivative terms should be negligible according to power-counting arguments. These results cast doubts on the quick convergence of the derivative expansion in the context of the induced gravity scenario.

However, for sufficiently small couplings the R^2 terms may leave the basic structure of a model unaffected. For example, in the case of one transverse dimension the induced $\hat{R}^2$ term on the brane does not change the large-distance behavior if its coefficient λ_2 obeys $\lambda_2 \ll (Mr_c)^3$. The bulk R^2 term does not affect the large-distance physics and becomes relevant only near the new threshold scale E_b^{-1} we discussed above. Thus the only modification to the model arises at the experimentally accessible submillimeter range, where new phenomena are expected.

We presented a string realization in which the R and R^2 terms in the bulk and on the brane are calculable. New threshold scales appear. The advantage of this model is that the physical behavior on both sides of the new scales can be predicted with confidence. In particular it can be seen that there are transitions which imply that the 5-dimensional Planck scale does not correspond to a real threshold.

In our study of induced gravity when the transverse space has more than one dimension we pointed out some interesting transitions in the nature of gravity on the brane. For example, in four large transverse dimensions without R^2 terms we found that there is a screening of gravity at distances larger than $r_0 = \Lambda r_c^2$, with Λ an ultraviolet cutoff. Such screening could be allowed at cosmological distances beyond the horizon. Taking $\Lambda \sim M$, $M_{Pl}^2 = M^6 r_c^4$ and $r_0 \sim 10^{26}$ m, we obtain $M \sim 5 \times 10^{-3}$ eV and $r_c \sim 10^{11}$ m. When an R^2 correction is included in the bulk with $L/r_c^2 \ll 1/r_c \ll 1/L \ll \Lambda$ we found that we have screening of gravity at distances larger than $r_0 \sim r_c^2/L$. Here also we can fit the Planck scale $M_{Pl} = M^3 r_c^2$ and choose r_0 to be larger than the cosmological horizon, while keeping L much smaller than a millimeter. It remains to be seen whether realistic models can be constructed along these lines.

The overall message of this work is that, in theories of induced gravity on a brane, new phenomena appear at threshold scales well below the higher dimensional PLanck energy scale. The physics above the new scales depends crucially on the structure of the fundamental theory.

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References

[1] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. **B485** (2000) 208, hep-th/0005016.

[2] G. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, hep-ph/0102216.

[3] C. Deffayet, G. Dvali, G. Gabadadze and A. Vainshtein, hep-th/0106001.

[4] J. E. Kim, B. Kyae and H. M. Lee, hep-ph/9912344, hep-th/0004005;
J. E. Kim and H. M. Lee, hep-th/0010093.

[5] C. Charmousis, R. Emparan and R. Gregory, JHEP **0105** (2001) 026, hep-th/0101198.

[6] S. Dubovsky and V. Rubakov, hep-th/0105243.

[7] G. Dvali and G. Gabadadze, Phys. Rev. **D63** (2001) 065007, hep-th/0008054;
M.Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros and C. Wagner, hep-ph/0102172.

[8] M. Green, J. Schwarz and E. Witten, *Superstring Theory, vols 1, 2*, Cambridge University Press, 1987;
J. Polchinski, Cambridge University Press;
E. Kiritsis, *Introduction to Superstring Theory*, Leuven University Press, 1998, hep-th/9709062.

[9] G. Pradisi and A. Sagnotti, Phys. Lett. **B216** (1989) 59;
M. Bianchi and A. Sagnotti, Phys. Lett. **B247** (1990) 517.

[10] E. Gimon and J. Polchinski, Phys. Rev. **D54** (1996) 1667, hep-th/9601038.

[11] E. Kiritsis, *Duality and Instantons in String Theory*, in the Proceedings of the ICTP Spring School, 1999, hep-th/9906018.

[12] A. Hammou and F. Morales, Nucl. Phys. **B573** (2000) 335, hep-th/9910144.

[13] J. Harvey and G. Moore, Phys. Rev. **D57** (1998) 2323, hep-th/9610237.

[14] A. Gregori, E. Kiritsis, C. Kounnas, N. Obers, M. Petropoulos and B. Pioline, Nucl. Phys. **B510** (1998) 423, hep-th/9708062.

[15] E. Kiritsis, C. Kounnas, M. Petropoulos and J. Rizos, Nucl. Phys. **B483** (1997) 141, hep-th/9608034.

[16] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche, T. R. Taylor, Nucl. Phys. **B489** (1997) 160, hep-th/9608012.

[17] I. Antoniadis, E. Gava and K. Narain, Nucl. Phys. **B383** (1992) 93, hep-th/9204030;
E. Kiritsis and C. Kounnas, Nucl. Phys. [Proc Supp.] **41** (1995) 331, hep-th/9410212;
Nucl. Phys. **B442** (1995), 472, hep-th/9501020;
J. Harvey and G. Moore, Nucl. Phys. **B463** (1996) 315, hep-th/9510182.

[18] I. Antoniadis, E. Gava, K. Narain and T. Taylor, Nucl. Phys. **B407** (1993) 706, hep-th/9212045.

[19] C.P. Bachas, P. Bain and M.B. Green, JHEP 9905 (1999) 011; hep-th/9903210.